Deterministic Communication in Ad-Hoc Radio Networks with Large Labels

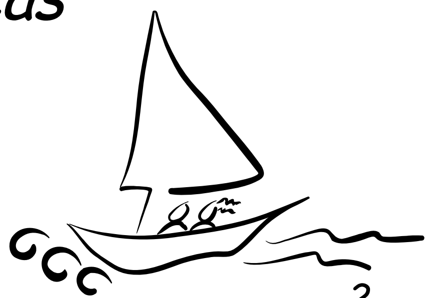
Aris Pagourtzis

National Technical University of Athens

With: Leszek Gasieniec and Igor Potapov
(*University of Liverpool)*

Ad-Hoc Radio Networks

- Expected to play an important role in future commercial and public interest applications.
- Suitable in situations where instant infrastructure is needed and no central system administration is available.
- Typical applications include:
  - *mobile communication in remote areas*
  - *tactical communications*
  - *disaster recovery situations*



Topology of Radio Network

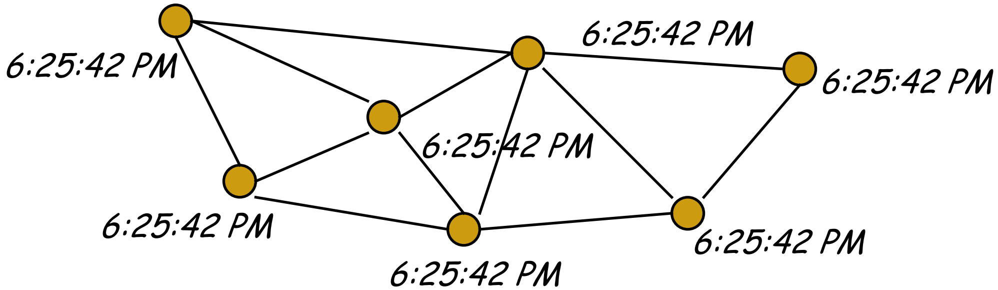
- Radio network is modelled as a graph

  $$G = (V, E)$$

  - V set of nodes, $|V|=n$
  - E set of connections (links)
- Graph G can be *directed* or *undirected*
- *Ad-hoc* stands for unknown topology

Communication Protocol

- Network is *synchronised*



- At any time step each node $v \in V$ is either in *transmitting* or *receiving* mode

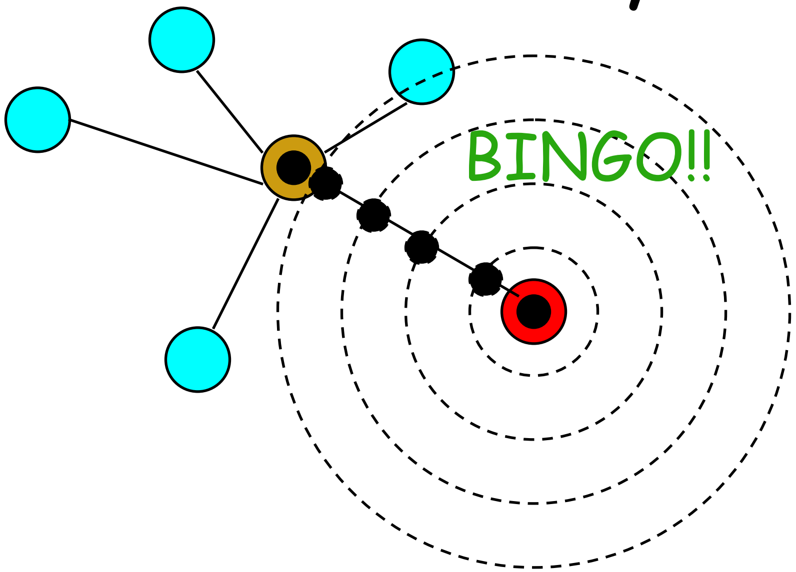
Communication Protocol

- *Transmission mode* – node v attempts to deliver possessed message m to all its neighbours
- *Receiving mode* – node v attempts to collect a message transmitted by one of its neighbours

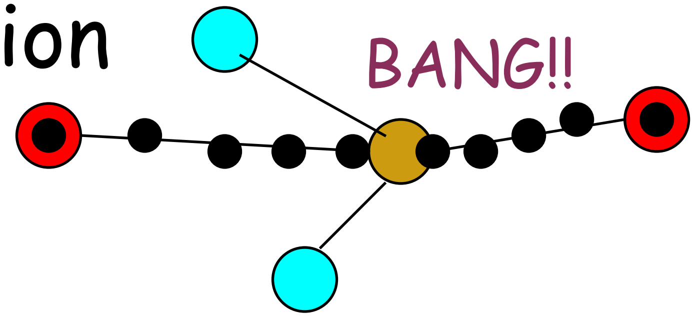
Warning: **If** more than one neighbour of v transmits **then** none of the messages reaches destination v

Communication Protocol

- Successful delivery



- Collision



Communication Primitives

- **Broadcasting:** *one-to-all communication;* a message *m* held originally in the source node *s* has to be delivered to all other nodes in the network
- **Gossiping:** *all-to-all communication;* total exchange of messages originated in every single node of the network

Crucial Parameters

- Number of nodes (size) *n*
- Largest label *N*

We are interested in time complexity!

Complexity Bounds (I)

- Lower: $\Omega(n \log n)$
- Upper: $O(n^2)$ using ROUND-ROBIN
  - if N not $O(n)$: $O(N^2)$

Complexity Bounds (II)

- Small labels: $N = O(n)$
  - Broadcasting: $O(n \log^2 n)$
  - Gossiping: $O(n^{3/2} \log^2 n)$

    Chrobak, Gasieniec, Rytter [FOCS'00]

- Large labels: $N = O(poly(n))$
  - Broadcasting:

$O(n^2)$ *Peleg [Manuscript'00]*

$O(n \log^2 n)$ *Chrobak et al. [FOCS'00]*

Complexity Bounds (III)

- Large labels: $N = O(poly(n))$

  Gossiping

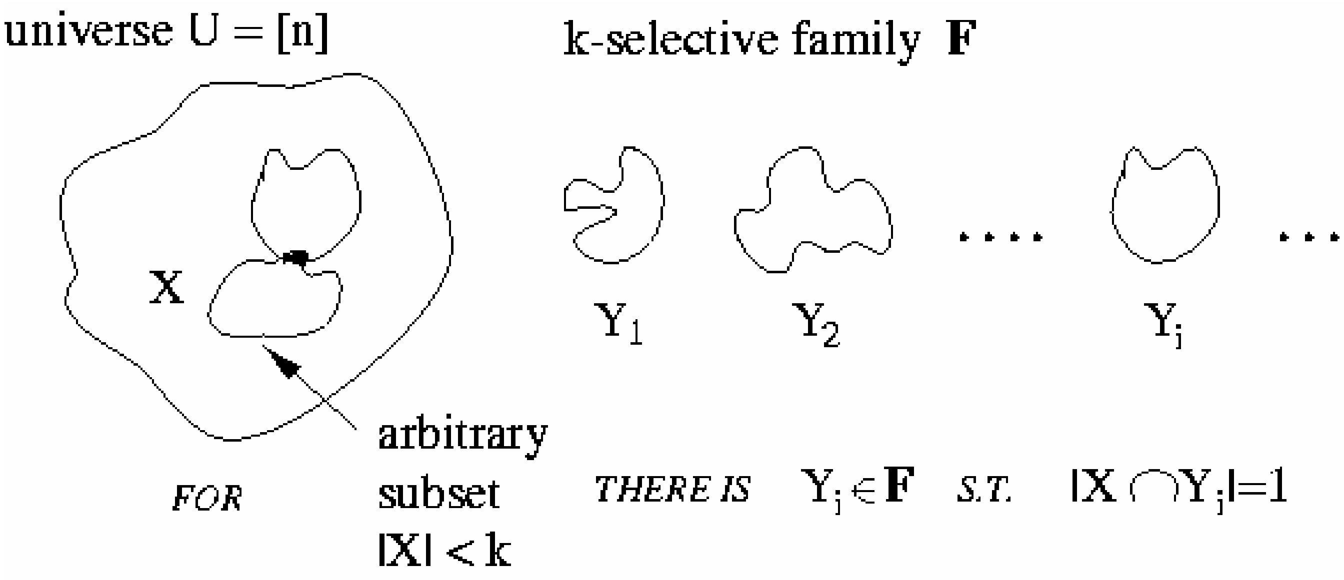
  - *Chrobak et al. [FOCS'00] :* $O(Nn^{1/2}\log^2 N)$

  - *Gasieniec, Pagourtzis, Potapov [ESA '02]:*

    In directed graphs: $O(n^{3/2}\log^3 n)$

    In undirected graphs: $O(n \log^3 n)$

k-selectivity



- A family **F** $= \{Y_1, Y_2, \ldots\}$ of subsets of some universe U is said to be *k-selective* for U iff for any $X \subseteq U$ with $|X| \le k$, there is a set $Y_j \in$ **F**, s.t. $|X \cap Y_j| = 1$.

k-selectivity

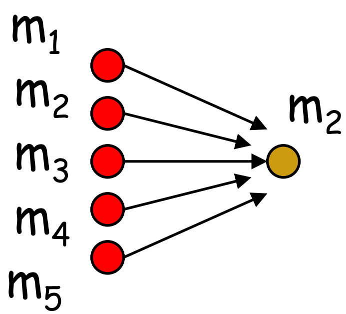
- A family **F** $= \{Y_1, Y_2, \ldots\}$ of subsets of some universe U is said to be *strongly k-selective* for U iff for any $X \subseteq U$, $|X| <= k$, and any $x \in X$ there is a set $Y_j \in$ **F**, s.t. $X \cap Y_j = \{x\}$.
- A family **F** $= \{Y_1, Y_2, \ldots\}$ of subsets of some universe U is said to be *linearly k-selective* for U iff for any $X \subseteq U$, $|X| <= k$, there exists set $X' \subseteq X$, s.t., $|X'| > k/2$ and for any $x \in X'$ there is a set $Y_j \in$ **F**, s.t., $X \cap Y_j = \{x\}$.

k-selectivity

U set of labels, U={1,2,…,N}

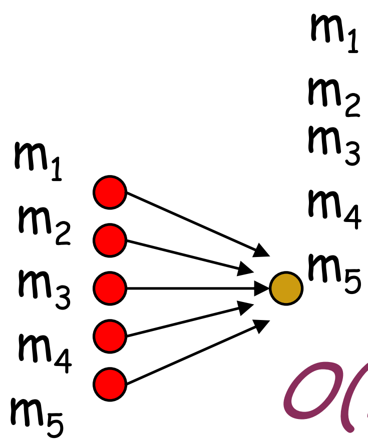
Size/Time

- Selectivity



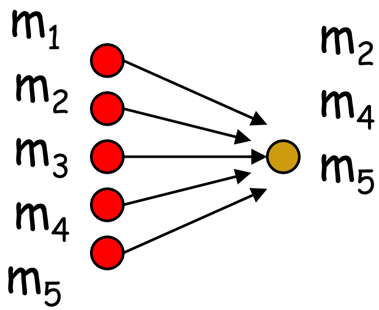
O(k log N)

- Strong selectivity



O(k² log N)

- Linear selectivity

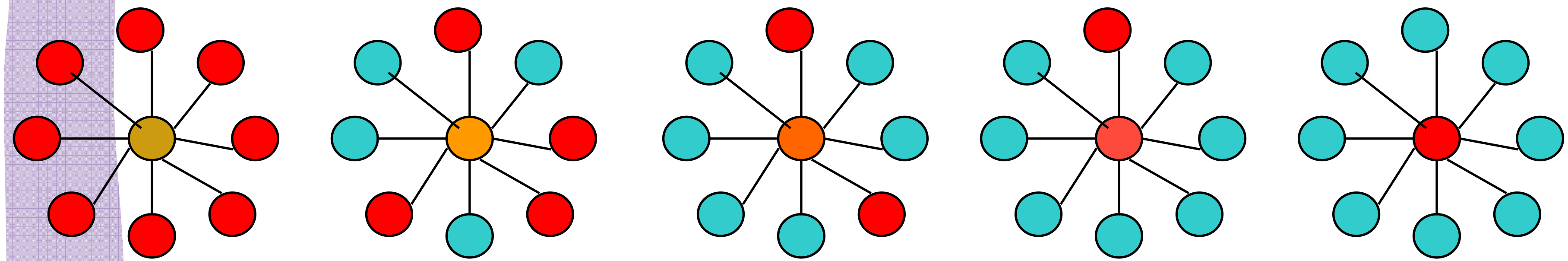


O(k log N)

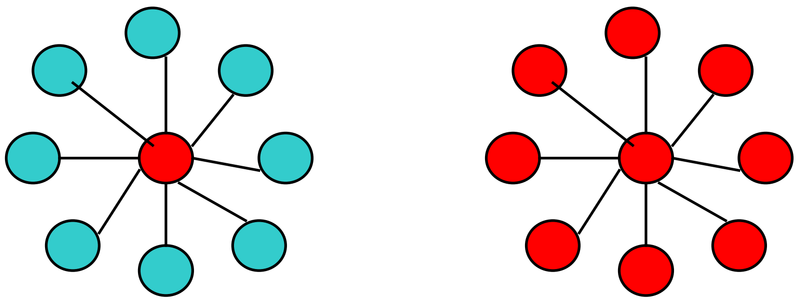
Gossiping in Stars $O(n \log n)$

use linearly selective family

- Collect branch messages in the centre

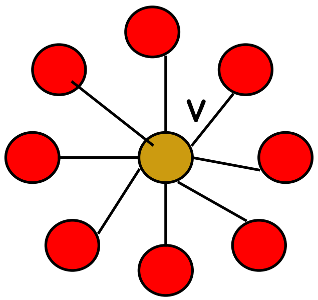


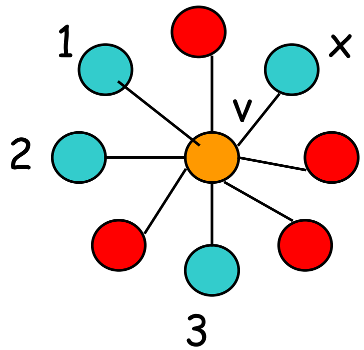
- Then distribute compound message



Approximate neighbour count

- Run linearly selective family once in order to learn about half the neighbourhood





$x \leq deg(v) \leq 2x$

Leader election

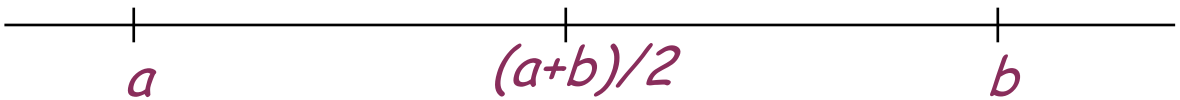
$O(B(n) \log(b-a))$

$O(n \log^3 n)$

- Binary search (e.g., for a node with the smallest label, the largest knowledge, etc.)
- Each step of binary search implemented by a vote based on broadcasting algorithm.

E.g., is there any node, s.t., $a <$ label $< (a+b)/2$?

- YES – further search between a and $(a+b)/2$
- NO – further search between $(a+b)/2$ and b



Partial Gossiping

+ decrement of (active) max-in-degree to k

Time

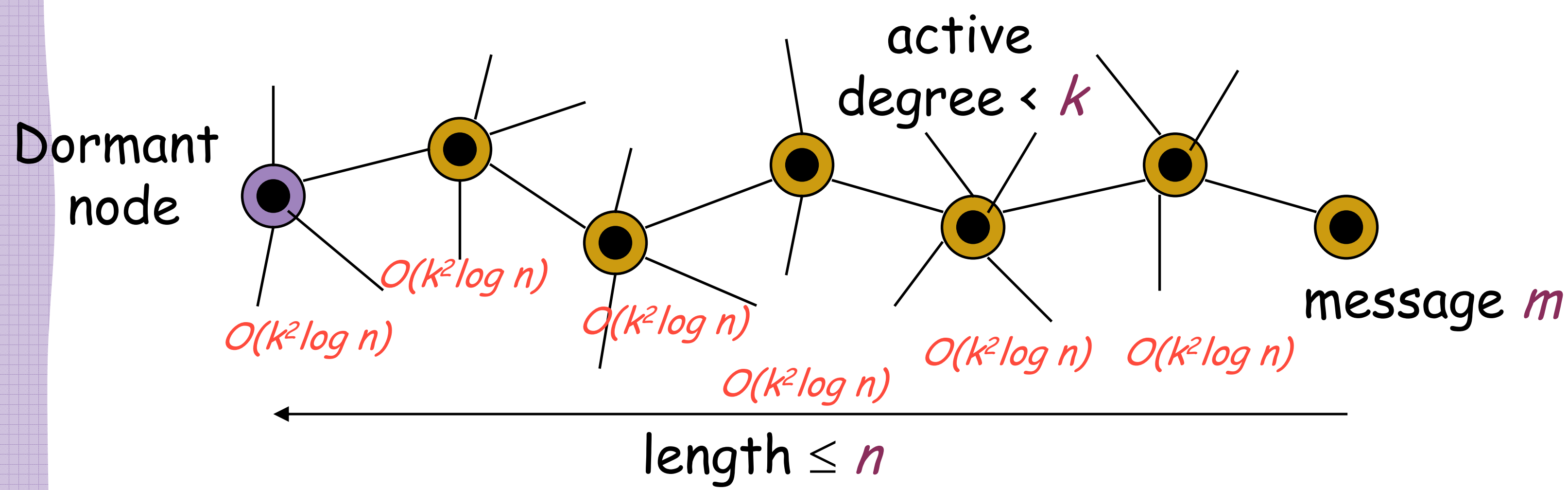
- Loop
  - Run N-linear selector: each node v learns about a fraction $A(v)$, (at least half), of its neighbourhood
  - Find v_{max}, s.t., $|A(v_{max})| \geq |A(v)|$, for all $v \in V$
  - Broadcast $A(v_{max})$ to all nodes — $O(n \log^3 n)$
  - For each $v \in V$, $A(v) = A(v) \setminus A(v_{max})$

  Until $|A(v_{max})| < k/2$ — $O(n^2 \log^3 n/k)$

Nodes whose original messages are already distributed become dormant (inactive)

Move remaining messages to dormant nodes

- Run strongly selective family with parameter k



Time $O(nk^2 \log n)$

Gossiping Algorithm

(in directed graphs)

Time

- Perform partial gossiping $O(n^2 \log^3 n/k)$
  - and decrement of active max-in-degree
- Move remaining messages to dormant nodes $O(nk^2 \log n)$
- *Repeat all transmissions performed in partial gossiping!* $O(n^2 \log^3 n/k)$

Total $O(n^{5/3} \log^3 n)$

where $k=n^{1/3}$

Gossiping Algorithm

(in directed graphs, with Tomasz Radzik, KCL)

- The gossiping algorithm can be tuned in the partial gossiping stage, s.t., a distance between any message and some dormant node is $\leq k$
- Then the second stage can be completed in time $O(k^3 \log n)$
- In total we get time complexity $O(n^{3/2} \log^3 n)$, with $k=n^{1/2}$

(Almost) best known bound in model with small labels!

Gossiping in Undirected Graphs

- Gossiping in connected graphs in model with known local neighbourhoods
  - *Find a leader λ in graph G (use binary search and broadcasting)*
  - *Perform DFS traversal of G starting at λ (collect all messages while visiting G)*
  - *Distribute compound message to all nodes in the network*

All done in time $O(n \log^3 n)$

Gossiping in Undirected Graphs

Time

- (1) Perform partial gossiping $O(n^2 \log^3 n/k)$
  - and decrement of active max-in-degree to $\leq k$
- (2.a) Run strongly selective family on active nodes only $O(k^2 \log n)$
  - this creates a number of connected components with entirely known neighbourhoods
- (2.b) Perform gossiping in each connected component $O(n \log^3 n)$
- (2.c) Repeat transmissions from stage (2.a)
- (3) Repeat transmissions from stage (1)

Total $(k = n^{2/3})$: $O(n^{4/3} \log^3 n)$

Gossiping in Undirected Graphs

- The gossiping algorithm can be speeded up by introduction of the *system of maps*
- This leads to almost linear $O(n \log^3 n)$ time gossiping in undirected graphs

The lower bound $\Omega(n \log n)$ is almost matched!

Conclusion

- The upper bounds in model with large labels (*$O(poly(n))$*) almost match bounds in model with small labels (*$O(n)$*)
  - Broadcasting $O(n \log^2 n)$
  - Gossiping in directed graphs $O(n^{3/2} \log^3 n)$
  - Gossiping in undirected graphs $O(n \log^3 n)$
- Similarly for even larger labels: $N=O(n^{\log^c n})$

  But: size of sets of selective families?
- What is the asymptotic complexity of gossiping in *ad-hoc* radio networks? In *known* radio networks?