Oblivious k-shot Broadcasting in Ad Hoc Radio Networks

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Motivation

- How can we accomplish communication in networks in an energy-efficient manner?
- We study this question for Radio Networks
- How do we measure energy efficiency?
- Assuming that the nodes transmit at a fixed power level:
  - Total number of transmissions
  - *The number of transmissions (shots) for each node*
- Give a limited budget of k shots to each node

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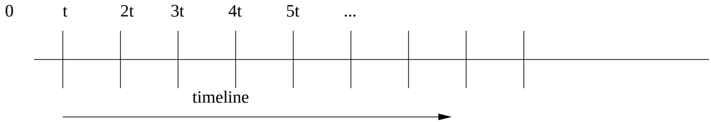
What is a Radio Network?

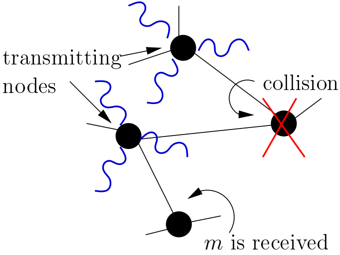
- A *directed* or *undirected* graph $G = (V, E)$
- Each node corresponds to a transmitter-receiver device
- Node v can send a message to node u *iff* $(v, u) \in E$
- Synchronization: the nodes transmit only at discrete time units called *steps*



- In each step, a node can be either in *receiving* or *transmitting* mode

Model of Communication

- **Radio Broadcast**: When node v transmits message m, m is delivered to all the neighbors of v



- **Collision**: If more than one neighbor of u transmits, then u receives no message and hears only noise

Broadcasting with k shots

We study protocols with the following characteristics

- Broadcasting (one-to-all communication):
  - A *source node* with a message m
  - m must be distributed to all the nodes of the network
- Unknown Topology: nodes only know the total number of nodes n
- Energy Efficiency: each node is limited to transmitting at most k times

Oblivious Protocols

Obliviousness: the decision of whether to transmit during the next step does *not* depend on the transmission history

- An oblivious protocol can be viewed as a sequence of *transmission sets*

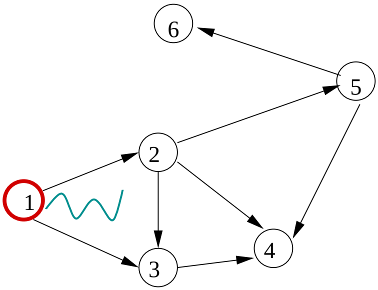
$$\{2, 3, 5\}, \{1\}, \{7, 1, 5, 2\}, \{4\}, \ldots$$

- When a node receives m for the first time, it transmits during the first k steps it appears in a transmission set

Oblivious Protocols: an Example

Example

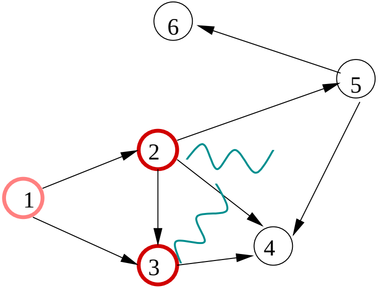
A broadcasting scheme $T = \{(1)\}$



Oblivious Protocols: an Example

Example

A broadcasting scheme $T = \{(1), (2, 3)\}$



Oblivious Protocols: an Example

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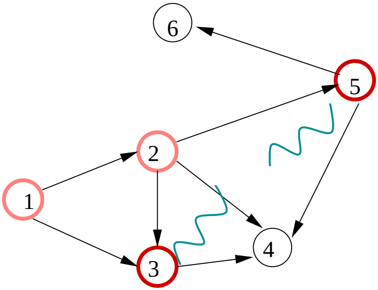

Oblivious Protocols: an Example

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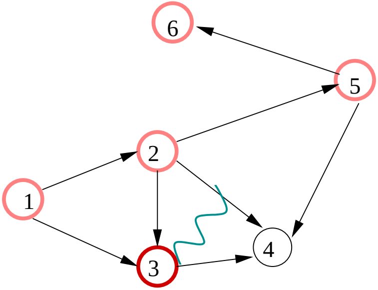

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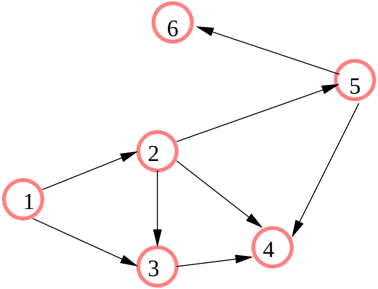

Related Work

Deterministic broadcasting with an unlimited number of shots

- Lower Bound: $\Omega(n \cdot \log D)$ [Clementi *et al.*, 2003]
- Upper Bound: $O(n \cdot \log n \cdot \log \log n)$ [De Marco, 2008]

Randomized broadcasting with an unlimited number of shots

- A matching Upper and Lower Bound: $\Theta(D \cdot \log(n/D) + \log^2 n)$ [Alon et al., 1991], [Czumaj and Rytter, 2003], [Kowalski and Pelc, 2003]

Related Work in k-shot

k-shot broadcasting in networks with *known* topology

- [Gasieniec *et al.*, 2008]
- [Kantor, Peleg *et al.*, 2009]
  Upper bound: $D + O(k \cdot n^{1/2k} \cdot \log^{2+1/k} n)$
  Lower bound: $D + \Omega(k \cdot (n - D)^{1/2k})$

Randomized *energy-efficient* broadcasting in unknown networks

- [Berenbrink *et al.*, 2009]

Our Results

Lower Bounds:

- Any k-shot ($k < n$) oblivious protocol needs $\Omega(\frac{n^2}{k})$ steps to complete broadcasting
- Any 1-shot *adaptive* protocol needs $\Omega(n^2)$ steps to complete broadcasting

Upper Bounds:

- There exists an oblivious algorithm which completes broadcasting in optimal $O(\frac{n^2}{k})$ time for $k \leqslant \sqrt{n}$ (and $O(n^{3/2})$ steps for $k > \sqrt{n}$)

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A Lower Bound for 1-shot

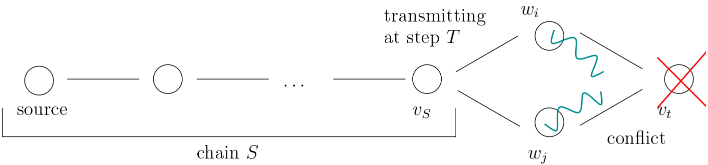
Theorem

Any 1*-shot protocol needs* $\Omega(n^2)$ *steps to complete broadcasting.*

- Given a 1-shot protocol, we build a chain S, where at the i-th node the transmission of m is delayed for at least $n - i$ steps
- Each node not in S must transmit alone in order to avoid a conflict
- Why is a conflict undesirable?

The Gadget

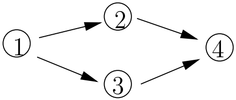
- Nodes w_i and w_j receive the message at step T from v_S



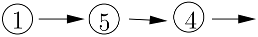
- A conflict occurs when nodes w_i and w_j transmit simultaneously
- Since v_t has no other neighbors, v_t never gets the message
- Broadcasting would never succeed in this graph

1-Shot Examples

- A non-valid protocol:
    - $\{1\}, \{2, 3\}, \{4\}, \{3\}, \{2\}, \ldots$



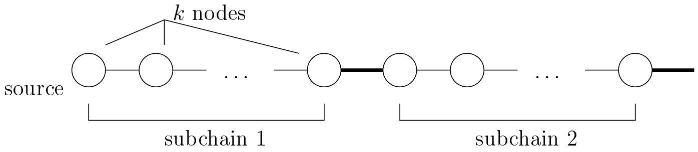
- A valid protocol (Lower Bound construction)
    - $\{1, 2, 3, 4, 5\}, \{2\}, \{3\}, \{5\}, \{4\}, \{1, 2\}, \{3, 4\}, \{5\}, \ldots$



The General Lower Bound

Theorem

Any oblivious k-shot ($k < n$) protocol needs $\Omega(\frac{n^2}{k})$ steps to complete broadcasting.



- Generalizing the argument for the 1-shot case
- We construct the chain from subchains of k nodes
- At the i-th subchain, the transmission of m is delayed for at least $n - i$ steps

A Matching Upper Bound

Theorem

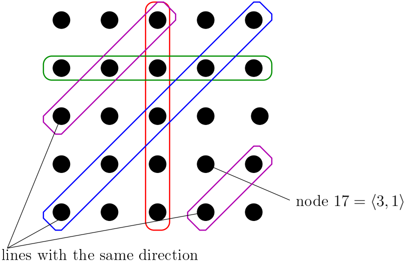
k-*shot broadcasting can be completed in* $O(\frac{n^2}{k})$ *steps, for* $k \leqslant \sqrt{n}$

The algorithm is based on the algorithm of [Chlebus *et al.*, 2000]

1. We define a family of sets (lines) with some nice properties
2. Based on lines, we construct the procedure Line-Transmit
3. We multiplex Line-Transmit with the standard Round-Robin procedure with the appropriate ratio

Defining Lines

- Let p be the smallest prime $\geqslant \sqrt{n}$
- Node i is mapped to the point $\langle i \text{ div } p, i \bmod p\rangle$



Definition

A *line* $L_{a,b}$ with direction a and offset b is the set

$$L_{a,b} = \begin{cases} \{\langle x, y\rangle : x \equiv b \pmod{p}\} & \text{if } a = p, \\ \{\langle x, y\rangle : y \equiv a \cdot x + b \pmod{p}\} & \text{else.} \end{cases}$$

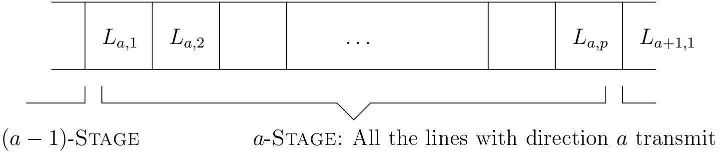
Properties of Lines

The following combinatorial properties hold:

1. Each line contains exactly p nodes
2. The total number of distinct lines is $p \cdot (p + 1)$
3. Each node belongs to $p + 1$ lines, one in each direction
4. There are p disjoint lines in each direction
5. Two lines of different directions have exactly one common node
6. For any two different nodes, there is exactly one line that contains both of them

LINE-TRANSMIT

Lines as transmission sets

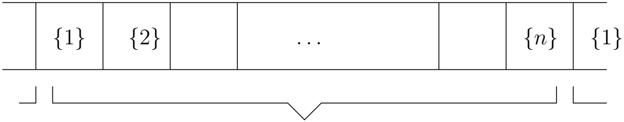


Procedure LINE-TRANSMIT

for $i = 1, 2, \ldots$ **do**
    for $a = 0, \ldots, p$ **do**
        a-STAGE
        for $b = 0, \ldots, p-1$ **do**
            Let all nodes in $L_{a,b}$ transmit
        end
    end
end

Round-Robin

The nodes transmit alone, one after the other



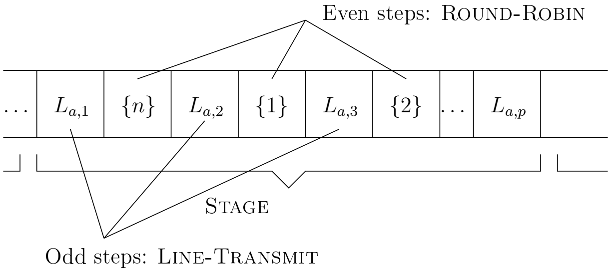
A round of Round-Robin

Procedure Round-Robin

for $i = 1, 2, \ldots$ **do**
    for $v = 1, 2, \ldots, p^2$ **do**
        Let node v transmit
    end
end

A First Mixing

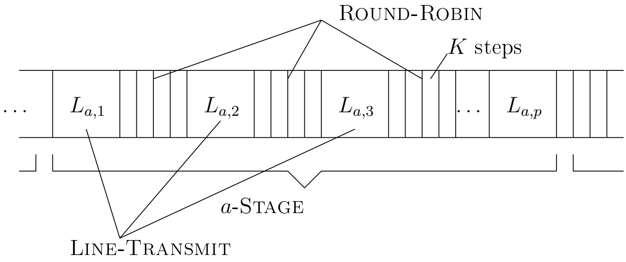
- Multiplex Round-Robin with Line-Transmit



- A node v starts transmitting only after receiving m
  1. Transmit for $k-1$ times at the *odd* steps where v belongs to the transmitting line
  2. Transmit once at the first *even* step with transmission set $\{v\}$

A Better Mixing

- The previous algorithm is not oblivious
- Solution: Increase the ratio of Round-Robin steps to Line-Transmit steps to $K = p/k$



Algorithm 3: Oblivious k-Shot

for *step* $i = 1, 2, \ldots$ **do**
  perform a Line-Transmit step ;
  perform K Round-Robin steps ;
end

Remarks

- The proof uses amortized analysis
- Our main contribution was to make the analysis flow when the k-shot restriction holds
- Corollary: The algorithm of [Chlebus *et al.*, 2000] has the same time performance when we restrict the number of shots to $\sqrt{n}$
- We use the $\sqrt{n}$-shot algorithm to yield an $O(n^{3/2})$-time protocol for k-shot broadcasting with $k > \sqrt{n}$

Conclusions

- We provide the first results on deterministic k-shot broadcasting in radio networks of unknown topology
- We prove an energy-time tradeoff in broadcasting algorithms

$$\#\text{shots} \times \#\text{steps} = \begin{cases} \Theta(n^2) & \text{for } k \leqslant \sqrt{n}, \\ \Omega(n^2) & \text{and} \\ O(k \cdot n^{3/2}) & \text{for } k > \sqrt{n}. \end{cases}$$

- Open Questions
  - match the lower bound for $k > \sqrt{n}$
  - Lower bounds for *adaptive* k-shot broadcasting protocols

Thank You