An overview of approximation algorithms for the Distance Constraint Vehicle Routing

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Topics covered in the presentation

- Definition and properties of Distance Constraint Vehicle Routing Problem **DVRP**
- A $3-$approximation algorithm for the unrooted DVRP by [1]
- Bicriteria approximation algorithm by [2]

Definition of DVRP

Input

- A complete graph $G = (V, E)$
- A metric distance $d : E \to \mathbb{R}^+$
- A starting position (depot, root) r
- A bound on the allowed length of a tour D

Output

A set of tours starting from r, with length at most D with the minimum cardinality (C), for which all vertices belong to at least one tour.

Definition: Unrooted DVRP (or minimum path cover) is a DVRP where the goal is to find the minimum cardinality set of paths (i.e. start and end location of every route is not the same) covering all vertices.

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Properties of DVRP

DVRP is NP-hard

Decision TSP (DTSP) can be reduced to the decision version of DVRP, where there exists a tour covering V with length at most D if and only if there exists a set of tours from r with length at most D with cardinality at most 1.

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A $3-$approximation algorithm for the unrooted DVRP [1], Intuitions

- The minimum cardinality (k) is guessed. This can be done, since possible results are $1, 2, \ldots, n = |V|$(polynomially bounded by the input), so exhaustive search can be applied.
- If $t_1, t_2, \ldots, t_k$ constitute a solution to DVRP, then these constitute k connected components of G. So, the minimum k connected components $C_i, i = 1, 2, \ldots, k$ have $\sum l(C_i) \leq \sum l(t_i) \leq k \cdot D$
- By doubling each edge in the connected components k Eulerian paths $p_1, p_2, \ldots, p_k$ can be created. They have total length $\sum l(p_i) \leq 2 \cdot \sum l(C_i) \leq 2 \cdot k \cdot D$.
- Each p_i can be cut into subpaths $p_{i1}, p_{i2}, \ldots, p_{il_i}$, where $p_i = p_{i1} * p_{i2} * * * p_{il_i}$ with length at most D and $l_1 + l_2 + \cdots + l_k \leq 3 \cdot k$, since $l_i \leq \frac{l(p_i)}{D} + 1$

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A $3-$approximation algorithm for the unrooted DVRP [1], Sketch of the algorithm

For every possible minimum cardinality $(k = 1, 2, \ldots, n)$

1. Compute the k minimum connected components $C_1, C_2, \ldots, C_k$ (using the Kruskal's algorithm for minimum spanning tree)
2. For each component C_i double its edges and compute an eulerian path p_i
3. Cut each $p_i, i = 1, 2, \ldots, k$ into segments $p_{i1}, p_{i2}, \ldots p_{il_i}$ that have length at most D with $p_i = p_{i1} * p_{i2} * \cdots p_{il_i}$. Call $S_i^k = \{p_{i1}, p_{i2}, \ldots p_{il_i}\}$
4. $S_k = \cup S_i^k$

Return the S_k with the minimum cardinality

Bicriteria approximation algorithm for DVRP [2]

Theorem: There is a $O(\log \frac{1}{\epsilon}, 1+\epsilon)$ bicriteria approximation algorithm for DVRP. (For $0 < \epsilon < 1$ if each tour is allowed to have length at most $(1+\epsilon) \cdot D$, then a set of tours containing V with cardinality at most $\log \frac{1}{\epsilon}$ times an optimal solution can be found.)

Bicriteria approximation algorithm for DVRP [2], Intuitions

- The set of vertices V, is partitioned into $1 + \lceil \log \frac{1}{\epsilon} \rceil$ subsets $V_0, V_1, \ldots, V_{\lceil \log \frac{1}{\epsilon} \rceil}$ according to their distance from the depot. Specifically,
  $V_0 = \{v : (1-\epsilon)\frac{D}{2} < d(r,v) \leq \frac{D}{2}\}$ and
  $V_j = \{v : (1-2^j\epsilon)\frac{D}{2} < d(r,v) \leq (1-2^{j-1}\epsilon)\frac{D}{2}\}, j = 1, 2, \ldots, \lceil \log \frac{1}{\epsilon} \rceil$
- If there is a path $P(v_1, v_2, \ldots, v_k) \subseteq V_j$ with $l(P) \leq 2^{j-1}\epsilon D$ then the tour $r * P * r = (r, v_1, v_2, \ldots, v_k, r)$ has length at most $d(r, v_1) + l(P) + d(v_k, r) \leq (1+\epsilon) \cdot D$.
- Let a tour $t = (r, u_1, u_2, \ldots, u_k, r)$ belonging to a solution of DVRP, then the restriction of t in V_j, $t_{V_j} = (u_{m_1}, u_{m_2}, \ldots, u_{m_l}) \subseteq V_j$, $m_1 < m_2 < \cdots < m_l$ has length less than $2^j \cdot \epsilon \cdot D$. Furthermore, t_{V_j} can be cut into two paths with length less than $2^{j-1} \cdot \epsilon \cdot D$.So, there are at most $2 \cdot \mathrm{OPT}$ paths covering V_j bounded by $2^{j-1} \cdot \epsilon \cdot D$.
- For each $j = 0, 1, \ldots, \lceil \log \frac{1}{\epsilon} \rceil$, at most $6 \cdot \mathrm{OPT}$ paths covering V_j can be found, with length at most $2^{j-1} \cdot \epsilon \cdot D$, applying the $3-$approximation algorithm described. So, at most $6 \cdot \mathrm{OPT} \cdot (1 + \lceil \log \frac{1}{\epsilon} \rceil)$ tours bounded by $(1+\epsilon) \cdot D$ covering V are created.

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Bicriteria approximation algorithm for DVRP [2], Sketch of the algorithm

1. Partition V in $V_0, V_1, \ldots, V_{\lceil \log \frac{1}{\epsilon} \rceil}$
2. For each V_j calculate a set of paths P_j in it, bounded by $2^{j-1} \cdot \epsilon \cdot D$ using the $3-$approximation algorithm described above
3. Take $P = \cup P_j$
4. Then the set of tours is $T = r * P * r = \{r * p * r | p \in P\}$

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