Selective Families, Superimposed Codes and Broadcasting on Unknown Radio Networks

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Introduction

- A radio network is a set of radio stations that are able to communicate by transmitting and receiving radio signals.

- A radio network can be modeled as a directed graph $G(V,E)$ where an edge (u,v) exists if and only if u can communicate with v in one hop.

Introduction

- Broadcast operation. It consists of transmitting a message from one source node to all the nodes.
- Two kinds of broadcast protocols. Spontaneous, non spontaneous.
- Completion time, termination time of a broadcast protocol.

Introduction

- In this paper we focus on the completion and termination time of (both spontaneous and non spontaneous) *Deterministic Distributed Broadcast* (DDB) protocols as a function of the following parameters of the network: the number *n* of nodes, the maximum in-degree *d* and the maximum eccentricity *D* over all possible source nodes.

Previous works

- Lower bound $\Omega(n)$ on the completion time of any DDB protocol running on a family of unknown, symmetric radio networks of diameter 3. [Bar- Yehuda]
- They also provide O((D+ logn) logn) for randomized protocol.
- $\Omega(\log^2 n)$ for randomized protocols even for graphs of constant eccentricity.
- The best known general lower bound for randomized protocols is $\Omega(D \log (n/D))$ [Kushilevitz, Mansour 1993]

Previous works

- For non spontaneous DDB protocols, Brushi and Del Pinto obtained a lower bound $\Omega(D \log n)$ for symmetric n node networks of diameter D.
- An equivalent lower bound for spontaneous DDB protocols has been proved by Chlebus.
- Chrobak, by using a variant of selective families, obtained an upper bound $O(n \log^2 n)$ which is almost optimal for general networks.

Results / Broadcast: lower bounds

DEFINITION: Let [n] = {1,…,n} and let $k \le n$. A family F of subsets of [n] is (n,k)-selective if, for every non empty subset Z of [n] s.t. $|Z| \le k$, there is a set F in F s.t. $|Z \cap F| = 1$.

THEOREM: For any DDB protocol P, for any n and for any $D \le n/6$, there exists an n-node directed graph G^P of maximum eccentricity D s.t. P completes broadcasting on G^P in $\Omega(n \log D)$ time-slots.

THEOREM: Let P be a DDB protocol. Then, for any n, for any $D \le n/6$, and for any $d \le n/D$, there exists an n-node directed graph G^P of maximum eccentricity D and in-degree bounded by d s.t. P completes broadcasting on G^P in $\Omega(Dd \log(n/d))$ time-slots.

Results / Broadcast: upper bounds

- The obtained DDB protocols have a completion time that does not contain *n* as linear factor but only D and d.

1. A DDB protocol SELECT-BROAD-A(n,d) that completes broadcasting in O(Dd logn) time-slots.
2. A DDB protocol SELECT-BROAD-B(n) that completes broadcasting in $O(Dd \log^2 n)$ time-slots.
3. For any positive constant a>0, a DDB protocol SELECT-BROAD $^{(a)}$ that completes broadcasting in $O(Dd \log^{2+a} n)$ time-slots.

Results / multibroadcast: upper bounds

DEFINITION: Let $k \leq n$. A family $\mathcal{F}$ of subsets of [n] is (n,k)-strongly-selective if, for every non empty subset Z of [n] s.t. $|Z| \leq k$ and for every element $z \in Z$, there is a set F in $\mathcal{F}$ s.t. $Z \cap F = \{z\}$.

1. A multibroadcast protocol SELECT-ALL-A(n,d) that has completion time $O(Dd^2 \log n)$ and termination time $O(nd^2 \log n)$.
2. A multibroadcast protocol SELECT-ALL-B(n) that has completion time $O(Dd^2 \log^2 n)$ and termination time $O(n^2)$.
3. For any positive constant a>0, there exists a multibroadcast protocol SELECT-ALL $^{(a)}$ that has completion time $O(Dd^2 \log^{2+a} n)$.

Results / multibroadcast: lower bounds

THEOREM: Let $\mathcal{F}$ be an (n,k)-strongly-selective family.

i. If $2 \le k \le \sqrt{2n} - 1$ then it holds that

   $|\mathcal{F}| \geq (k^2 / 16 \log k) \log n$.

ii. If $k \geq \sqrt{2n}$ then it holds that $|\mathcal{F}| \geq n$.

Adopted techniques

The new broadcast technique overcomes two technical difficulties:

1. How to achieve a completion time that does not contain *n* as linear factor.
2. How choosing the correct selective family when d and n are not known by the nodes.

Connection between selective families and radio broadcasting

- An oblivious DDB protocol on unknown networks of n nodes and maximum in-degree k can be represented by a binary matrix M with n columns (i.e. the nodes) and each row corresponding to a time-slot. The entry $M_{t,i} = 1$ iff node i may transmit in time-slot t.

  Necessary condition: for any subset of at most k columns, there exists a row with a single 1 in the given columns.

DDB protocols

Definition. A *Deterministic Distributed Broadcast* (DDB) protocol P is a protocol that works in time-slots (numbered 0,1,…) according to the following rules:

1. In the initial time-slot a specified node (i.e. the source) transmits a message (called the initial message)
2. In each time-slot, each node either acts as transmitter or as receiver or is non active.
3. A node receives a message in a time-slot if and only if it acts as receiver and exactly one of its in-neighbors acts as transmitter in that time-slot.
4. The action of a node in a specific time-slot is a function of its own label, the number of the current time-slot t and the message received during the previous time-slots.

SELECT-BROAD-A(n,d)

The protocol uses an (n,d)-selective family and assumes the knowledge of d and *n*.

- Set all nodes to the active state, and let *s* transmit the initial message.
- After the first time-slot, turn *s* to the non active state.
- Perform a sequence of consecutive *phases*. Each *phase* consists of |F | time slots. At time-slot j of *phase* i, each active node v acts according to the following rule: v transmits the initial message along its outgoing edges if and only if

1. The label of v belongs to the j-th set of F, and
2. v has received the initial message before the beginning of *phase* i.

SELECT-BROAD-A(n,d)

- THEOREM: protocol SELECT-BROAD-A(n,d) completes broadcasting (and terminates) in O(Dd logn) time-slots on any n-node graph of maximum eccentricity D and in degree bounded by d.

- Claim: A node v receives the initial message at phase i of protocol SELECT-BROAD-A(n,d) if and only if v is at distance i+1 from the source i.

SELECT-BROAD-B(n)

The protocol assumes the knowledge of the size of the network.

- Each node runs a sequence of phases, each of them consisting of logn time-slots.
- In time slot l ($1 \leq \ell \leq \log n$) of phase h, each node runs time-slot h of SELECT-BROAD-A($n,2^{\ell}$).

THEOREM: Protocol SELECT-BROAD-B(n) completes broadcasting and terminates in $O(Dd \log^2 n)$ time-slots on any n-node graph G with maximum eccentricity D and maximum in-degree d.

SELECT-BROAD $^{(a)}$

- Consider the following family of functions:

  $f_0^a(z)=0$, $f_k^a(z)= 2^{k(2/a)}(k-z)$, $k=1,2,3,\dots$