## Combinatorial Auctions

Best algorithms and bounds

Looking for algorithms with:

- good approximation ratio
- Polynomial time / number of value queries
- Truthful?

Randomization over deterministic truthful mechanisms

Every (universally) truthful  $m^{\frac{1}{2}-\epsilon}$  —approximation mechanism with submodular bidders makes exponentially many value queries. [Dobzinsky 2011] Every (universally) truthful  $m^{\frac{1}{2}-\epsilon}$  —approximation mechanism with submodular bidders makes exponentially many value queries. [Dobzinsky 2011]

 Truthfulness ↔ each bidder faces a *menu* where each bundle has a price and is assigned the most profitable bundle (the taxation principle)

Possible allocations and prices he could get by the algo

depending on the other bidders.

- In every algorithm with good approximation ∃ instance in which at least one bidder faces an exponentially large menu
- # value queries to find profit maximizing bundle = size of the menu

е	[V08]
$\overline{e-1}$	

Class	Queries	Approx	IC approx	Lower bound
$\mathbf{Gen}$	Any	$\sqrt{m}$	$\frac{m}{\sqrt{\log m}}$ $\sqrt{m}$ (rand)	$m^{\frac{1}{2}-\epsilon}$ Section 1.6, [NS06]
	Value	$\frac{m}{\sqrt{\log m}}$	$\frac{m}{\sqrt{\log m}}$ [HKDMT04]	$\frac{m}{\log m}$ [BN05a, DS05]
	Demand	$\sqrt{m}$ [BN05a]	$\frac{m}{\sqrt{\log m}}$	$m^{\frac{1}{2}-\epsilon}$
			$\sqrt{m}$ (rand) [LS05, DNS06]	
SubA	Value	$\sqrt{m}$	$\sqrt{m}$ [DNS05]	$m^{\frac{1}{4}}$
	Demand	2  (rand) [Fei06]	$\sqrt{m}$	2 [DNS05]
xos	Value	$\sqrt{m}$	$\sqrt{m}$	$m^{\frac{1}{4}}$ [DS06]
	Demand	2 [DNS05] $\frac{\epsilon}{e-1}$ (rand) [Fei06]	$\sqrt{m}$ $\log^2 m$ (rand) [DNS06]	$\frac{e}{e-1}$ [DNS05]
SubM	Value	▶ 2 [LLN06]	$\sqrt{m}$	$\frac{e}{e-1}$ [KLMM05]
	Demand	2	$\sqrt{m}$	276 275 [FV06]
		$\frac{e}{e-1}$ -10 <sup>-4</sup> (rand) [FV06]	$\log^2 m$ (rand)	
Subs	Value	1 [Ber05]	1	
	Demand	1 [GS99, BM97]	1	
k <b>D</b> up	Demand	$m^{\frac{1}{k+1}}$ [BKV05, DS05]	$k \cdot m^{\frac{1}{k-2}}$ [BGN03]	$m^{\frac{1}{k+1}-\epsilon}$ [BGN03, DS05]
Proc	Any	$\ln n$ [NS06]	-	$\log n$ [Nis02]

## $m^{\frac{1}{2}-\epsilon} [D11]$ (truthful) $m^{\frac{1}{3}-\epsilon} [DSS15]$ (truthful)

## Truthful $O(\sqrt{m})$ –approximation for subadditive bidders [DNS05]

- 1. Query each bidder *i* for  $v_i(M)$  and  $v_i(j)$ ,  $\forall j \in M$ .
- 2. Construct bipartite graph between bidders and items and compute the maximum weighted matching *P*.

 $\begin{array}{c} \bullet_{i} \\ \forall i \in N \end{array} \quad v_{i}(j) \quad \forall j \in M \end{array}$ 

- 3. Allocate the items according to P, unless  $\max_{i} v_i(M) = v_k(M)$  is higher than the value of P. In this case, give all the items to k.
- 4. Let each bidder pay her VCG price.

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✓  $\sqrt{m}$  – approx.

Polynomial

time

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