Mechanism Design

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Mechanism Design

Designing algorithms for settings where inputs are controlled by selfish agents.

Center

A *center* wants to implement some function of the inputs

Agents

Agents have preferences over possible outcomes and may lie about their inputs if it is profitable to do so.

Mechanism Design

Definitions

- there is a set O of the possible *outcomes* of the mechanism

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- the set of all types is the domain D
- the agents may report any type $y \in D$
- the center wants to *implement* a *social choice function* $f : D^n \to O$, where n is the number of agents

Single Selfish Agent [A. Archer, R. Kleinberg EC' 08]

Mechanism design for a single agent assumes the agent seeks to use a best response to the enviroment of mechanism and aims to ensure that truth-telling is the best responce regardless of the agent's type. For multi-agent mechanism design, various game-theoretic solution concepts are studied, the most common being dominant strategy equilibrium and Bayes-Nash equilibrium. All of these solution concepts also assume tha each agent in isolation seeks to use best response to the environment of the mechanism, but they differ in how the presence of the other players defines that environment.

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Mechanism Design

Implementation of a social choice function
without money

Definition

A mechanism – social choice function $g : D \to O$ – is said to *implement* a social choice function f if for each $x \in D$ there exists a $y \in D$ such that :

$$g(y) = f(x)$$

$$x(g(y)) \geq x(g(z)) \quad \forall z \in D$$

Mechanism Design

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Definition

A social choice function f is said *implementable* if there exists a mechanism g that implements it.

Mechanism Design

Truthful implementation of a social choice function
without money

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A mechanism – social choice function $g : D \to O$ – is said to *truthfully implement* a social choice function f if for each $x \in D$:

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Definition

A social choice function f is said *truthfully-implementable* if there exists a mechanism g that truthfully-implements it.

Representation by a Graph

We define a graph $G_f(V,\ V^2,\ w)$ from a social choice function f as follows :

$$V = D$$

$$w(x, y) = x(f(x)) - x(f(y))$$

Representation by a Graph

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$$V = D$$

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Without money

Condition for truthfulness

A social choice function f is truthfully-implementable without money if and only if G_f has *no negative-edge*.

We care only about the sign of the edges!

Revelation Principle

Revelation Principle

A social choice function f is implementable if and only if it is truthfully-implementable.

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Strategic Voting

Definition

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- agent i is described with by an order of the element's of A denoted by $\prec_i$ which is called i's preference order. When $a \prec_i b$ for some $a, b \in A$, we say that agent i prefers b to a. Let L be the set of all linear orders of A.

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- the center wants to implement a social choice function – voting rule $f : L^n \rightarrow A$ or
- the center wants to implement a social welfare function $F : L^n \rightarrow L$.

Strategic Voting

Useful Properties of Voting Rules

Unanimity

A social welfare function F satisfies *unanimity* if for every $\prec \in L$ $F(\prec, \ldots, \prec) = \prec$. That is, if all voters have identical preferences then the social preference is the same.

Strategic Voting

Useful Properties of Voting Rules

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Dictatorship for social welfare functions

Agent i is a *dictator* in social welfare function F if for all $\prec_1, \ldots, \prec_n \in L$ $F(\prec_1, \ldots, \prec_n) = \prec_i$. The social preference in a *dictatorship* is simply that of the dictator, ignoring all other voters. F is not a dictatorship if no ii is a dictator in it.

Strategic Voting
Useful Properties of Voting Rules

Independence of irrelevant alternatives

A social welfare function satisfies *independence of irrelevant alternatives* if the social preference between any two alternatives a and b depends only on the voters' preferences between a and b. Formally, for every $a, b \in A$ and every $\prec_1, \ldots, \prec_n, \prec'_1, \ldots, \prec'_n \in L$, if we denote $\prec = F(\prec_1, \ldots, \prec_n)$ and $\prec' = F(\prec'_1, \ldots, \prec'_n)$ then $a \prec_i b \Leftrightarrow a \prec'_i b$ for all i implies that $a \prec b \Leftrightarrow a \prec' b$.

Strategic Voting

Arrow's theorem [Arrow 1951]

Every social welfare function over a set of more than 2 candidates ($|A| \geq 3$) that satisfies unanimity and independence of irrelevant alternatives is a dictatorship.

Strategic Voting

Proof of Arrow's theorem [Nisan '07]

Claim (pairwise neutrality)

Let $\prec_1, \ldots, \prec_n$ and $\prec'_1, \ldots, \prec'_n$ be two player profiles such that for every player i, $a \prec_i b \Leftrightarrow c \prec'_i d$. Then $a \prec b \Leftrightarrow c \prec' d$ where $\prec = F(\prec_1, \ldots, \prec_n)$ and $\prec' = F(\prec'_1, \ldots, \prec'_n)$.

Strategic Voting

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Proof.

We merge each $\prec_i$ and $\prec'_i$ into a single preference $\prec_i$ by putting c just above a and d just below b and preserving the internal order within each of the pairs (a, b) and (c, d). Now using unanimity, we have that $c \prec a$ and $b \prec d$, and by transitivity $c \prec d$. □

Strategic Voting

Proof of Arrow's theorem [Nisan '07]

Proof of Arrow's theorem.

Take any $a \neq b \in A$, and for every $0 \leq i \leq n$ define a preference profile π_i in which exactly the first i players rank a above b, i.e., in pi_i $a \prec_j b \Leftrightarrow j \leq i$ (the exact ranking of the other alternatives does not matter). By unanimity, in $F(\pi_0)$, we have $b \prec a$, while in $F(\pi_n)$ we have $a \prec b$. By looking at $\pi_0, \ldots, \pi_n$ at some point the ranking between a and b flips, so for some i^* we have that in $F(\pi_{i-1})$, $b \prec a$, while in $F(\pi_i)$, $a \prec b$. We conclude the proof by showing that i^* is a dictator. □

Strategic Voting

Proof of Arrow's theorem [Nisan '07]

Claim

Take any $c \neq d \in A$. If $c \prec_{i^*} d$ then $c \prec d$ where $\prec = F(\prec_1, \ldots, \prec_n)$.

Proof.

Take some alternative e which is different from c and d. For $i < i^*$ move e to the top in $\prec_i$, for $i > i^*$ move e to the bottom in $\prec_i$, and for i^* move e so that $c \prec_{i^*} e \prec i^* d$ – using independence of irrelevant alternatives we have not changed the social ranking between c and d. Now notice that players' preferences for the ordered pair (c, e) are identical to their preferences for (a, b) in π_i, but the preferences for (e, d) are identical to the preferences for (a, b) in π_{i-1} and thus using the pairwise neutrality claim, socially $c \prec e$ and $e \prec d$, and thus by transitivity $c \prec d$. □

Strategic Voting

Useful Properties of Voting Rules

Dictatorship for social choice function

Voter i is a dictator in social choice function f if for all $\prec_1, \ldots, \prec_n \in L\ \forall b \neq a, a \succ_i b \Rightarrow f(\prec_1, \ldots, \prec_n) = a$. The social choice function f is called a dictatorship if some i is a dictator in it.

Strategic Voting

Useful Properties of Voting Rules

Dictatorship for social choice function

Voter i is a dictator in social choice function f if for all $\prec_1, \ldots, \prec_n \in L \; \forall b \neq a, a \succ_i b \Rightarrow f(\prec_1, \ldots, \prec_n) = a$. The social choice function f is called a dictatorship if some i is a dictator in it.

Onto

A social choice function f is said to be onto a set A if for every $a \in A$ there are $\prec_1, \ldots, \prec_n \in L$ such that $f(\prec_1, \ldots, \prec_n) = a$.

Strategic Voting

Theorem [Gibbard 1973], [Satterthwaite 1975]

Let f be an truthful social choice function onto A, where $|A| \geq 3$, then f is a dictatorship.

Strategic Voting

Proof of Gibbard-Satterthwaite [Nisan '07]

Definition

A social choice function f is monotone if $f(\prec_1, \ldots, \prec_i, \ldots, \prec_n) = a \neq a' = f(\prec_1, \ldots, \prec_i, \ldots, \prec_n)$ implies that $a' \prec_i a$ and $a \prec'_i a'$. That is, if the social choice changed from a to a' when a single voter i changed his vote from $\prec_i$ to $\prec'_i$ then it must be because he switched his preference between a and a'.

Proposition

A social choice function is incentive compatible if and only if it is monotone.

Strategic Voting

Proof of Gibbard-Satterthwaite [Nisan '07]

Notation

Let $S \subset A$ and $\prec \in L$. Denote by $\prec^S$ the order obtained by moving all alternatives in S to the top in $\prec$. Formally, for $a, b \in S$, $a \prec^S b \Leftrightarrow a \prec b$, for $a, b \in S$, also $a \prec^S b \Leftrightarrow a \prec b$, but for $a \in S$ and $b \in S$, $a \prec^S b$.

Definition

The social welfare function F that extends the social choice function f is defined by $F(\prec_1, \ldots, \prec_n) = \prec$, where $a \prec b$ if and only if $f(\prec_1^{\{a,b\}}, \ldots, \prec_n^{\{a,b\}}) = b$.

Strategic Voting
Proof of Gibbard-Satterthwaite [Nisan '07]

Claim
For any $\prec_1, \ldots, \prec_n \in L$ and any S, $f(\prec_1^S, \ldots, \prec_n^S) \in S$.

Strategic Voting

Proof of Gibbard-Satterthwaite [Nisan '07]

Claim

For any $\prec_1, \ldots, \prec_n \in L$ and any S, $f(\prec_1^S, \ldots, \prec_n^S) \in S$.

Proof.

Take some $a \in S$ and since f is onto, for some $\prec'_1, \ldots, \prec'_n$, $f(\prec'_1, \ldots, \prec'_n) = a$. Now, sequentially, for $i = 1, ..., n$, change $\prec'_i$ to $\prec_i^S$. We claim that at no point during this sequence of changes will f output any outcome $b \notin S$. At every stage this is simply due to monotonicity since b $\prec_i^S$ a for $a \in S$ being the previous outcome. □

Strategic Voting

Proof of Gibbard-Satterthwaite [Nisan '07]

Lemma 1

If f is an truthful social choice function onto A then the extension F is a social welfare function.

Strategic Voting

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If f is an truthful social choice function onto A then the extension F is a social welfare function.

Lemma 2

If f is an truthful social choice function onto A, which is not a dictatorship then the extension F satisfies unanimity and independence of irrelevant alternatives and is not a dictatorship.

Ways to Escape the Gibbard-Satterthwaite Impossibility

1. Restriction on the domain

Ways to Escape the Gibbard-Satterthwaite Impossibility

Restriction on the domain

Single Peaked Preferences

Moulin's characterization [Moulin 1980]

A social choice function f for the single peaked domain on the line is truthful, onto, and anonymous if and only if there exist $y_1, \ldots, y_{n-1} \in \mathbb{R}$ such that for all $p_i \in \mathbb{R}$,

$$f(p_1, \ldots, p_n) = \text{med}(p_1, p_2, \ldots, p_n, y_1, y_2, \ldots, y_{n-1})$$

Ways to Escape the Gibbard-Satterthwaite Impossibility

1. Restriction on the domain
2. Randomized social choice functions
3. Imposing Mechanisms

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1. Restriction on the domain
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4. Money
5. Verification

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Mechanism Design with money

Implementation of a social choice function

Definition

A mechanism – a pair $(g,\ p)$ with g a social choice function $g : D \to O$ and p a *payment function* $p : D \to \mathbb{R}$ – is said to *implement* a social choice function f if for each $x \in D$ there exists a $y \in D$ such that :

$$g(y) = f(x)$$

$$x(g(y)) + p(y) \geq x(g(z)) + p(z) \quad \forall z \in D$$

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Definition

A social choice function f is said *implementable*, with money, if there exists a mechanism (g, p) that implements it.

Mechanism Design

Truthful implementation of a social choice function

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Definition

A social choice function f is said *truthfully-implementable*, with money, if there exists a mechanism (g, p) that truthfully-implements it.

Revelation Principle

Revelation Principle

For both the case with and without money a social choice function f is implementable if and only if it is truthfully-implementable.

Representation by a Graph

We define a graph $G_f(V,\ V^2,\ w)$ from a social choice function f as follows :

$$V = D$$

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Condition for truthfulness without money

A social choice function f is truthfully-implementable without money if and only if G_f has *no negative-edge*.

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With money ?

Rochet's Theorem

Definition

A social choice function $f : D \rightarrow O$ satifies *cycle monotonicity (CMON)* if for every sequence of types $x_1, \ldots, x_k$ it holds that :

$$\sum_{i=0}^{k} x_{i+1}(f(x_{i+1})) - x_i(f(x_{i+1})) \geq 0$$

Rochet's Theorem

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$$\sum_{i=0}^{k} x_{i+1}(f(x_{i+1})) - x_i(f(x_{i+1})) \geq 0$$

Theorem

A social choice function f is truthfully-implementable with money if and only if satisfies CMON i.e. G_f has *no negative cycle.*

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Partial Verification

- In the usual mechanism design setting, an agent with type x can report any other type $y \in D$.

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Partial Verification

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- In the *partial verification model* the types that the agent can report is *limited* and may depend on u.

Definition

A *misreport correspondence* is a function $M : D \to 2^D$, which for each type u specifies the set of types $M(x) \subseteq D$ that the agent can possibly report.

Partial Verification

M−Implementation of a social choice function

Definition

A mechanism – a pair $(g, \; p)$ with g a social choice function $g : D \rightarrow O$ and p a payment function $p : D \rightarrow \mathbb{R}$ – is said to *M-implement* a social choice function f if for each $x \in D$ there exists a $y \in M(x)$ such that :

$$g(y) = f(x)$$

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Definition

A social choice function f is said $M-$*implementable*, with money, if there exists a mechanism (g, p) that $M-$implements it.

Partial Verification

Truthful M−Implementation of a social choice function

Definition

A mechanism – a pair $(g, \ p)$ with g a social choice function $g : D \to O$ and p a payment function $p : D \to \mathbb{R}$ – is said to *truthfully M-implement* a social choice function f if for each $x \in D$:

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Partial Verification

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Definition

A social choice function f is said *truthfully $M-$implementable*, with money, if there exists a mechanism $(g,\ p)$ that truthfully $M-$implements it.

Representation by a Graph in the case of Partial Verification

We define a graph $G_{M,f}(V,\ E,\ w)$ from a social choice function f and a misreport correspondense M as follows :

$$V = D$$

$$E = \{(x, y) \mid x \in D, \quad y \in M(x)\}$$

$$w(x, y) = x(f(x)) - x(f(y))$$

Representation by a Graph in the case of Partial Verification

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$$w(x, y) = x(f(x)) - x(f(y))$$

Without money

Condition for truthfulness

A social choice function f is truthfully $M-$implementable without money if and only if $G_{M,f}$ has *no negative-edge*.

Representation by a Graph in the case of Partial Verification

We define a graph $G_{M,f}(V,\ E,\ w)$ from a social choice function f and a misreport correspondense M as follows :

$$V = D$$

$$E = \{(u, v) \mid u \in D, \quad v \in M(x)\}$$

$$w(u, v) = u(f(u)) - u(f(v))$$

With money

Condition for truthfulness

A social choice function f is truthfully $M-$implementable without money if and only if $G_{M,f}$ has *no negative-cycle*.

Non-truthfull implementation

Example [J. Green, J. Laffort '86]

Consider a setting with $O = \{T, F\}$, $D = \{u, v, w\}$ and $x(T) = 1$, $x(F) = 0 \ \ \forall x \in D$. Suppose that the correspondence M is given by $M(u) = \{u, v\}$, $M(v) = \{v, w\}$, $M(w) = \{w\}$ and we would like to implement the social choice function $f(u) = F$, $f(v) = F$, $f(w) = T$. We can set $g(u) = g(v) = F, g(w) = T$: under this mechanism $g(u') = F \ \ \forall u' \in M(u)$ and v, w can both report w to obtain their preferred outcome $g(w) = T$.

Non-truthfull implenentation harndess

Definition of IMPLEMENTABILITY problem

Input : domain D, outcome set O, social choice function $f : D \rightarrow O$ and correspondence M.

Output : there exists an outcome function g that $M-$implements f ?

Non-truthfull implenentation harndess

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Theorem [Auletta et. al '11]

The IMPLEMENTABILITY problem is NP-hard.

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Output : there exists an outcome function g that $M-$implements f ?

Theorem [Auletta et. al '11]

The IMPLEMENTABILITY problem is NP-hard.

Similarly for implementation with money.

Partial Verification – Revelation Principle

Nested Range Conditions

A misreport corespondence M is said to satisfy *nested-range conditions (NRC)* if :

$$\forall u_1 \in D \; \forall u_2 \in M(u_1) \; (u_3 \in M(u_2) \rightarrow u_3 \in M(u_1))$$

Partial Verification – Revelation Principle

Nested Range Conditions

A misreport corespondence M is said to satisfy *nested-range conditions (NRC)* if :

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Theorem [J. Green, J. Laffort '86]

If a misreport correspondence M satisfy NRC then a social choice function f is $M-$implementable if and only if it is truthfully $M-$implementable.

Partial Verification – Revelation Principle

Nested Range Conditions

A misreport corespondence M is said to satisfy *nested-range conditions (NRC)* if :

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Theorem [J. Green, J. Laffort '86]

If a misreport correspondence M satisfy NRC then a social choice function f is $M-$implementable if and only if it is truthfully $M-$implementable.

Theorem [L. Yu '10]

A misreport correspondence M satisfy *Strong Decomposability* if and only if revelation principle holds.

Non-symmetric verification – Positive results

Examples of $M-$ truthful implementations

Scheduling [Auletta et al. '06] Agent can only report better speed.

Non-symmetric verification – Positive results

Examples of $M-$ truthful implementations

Scheduling [Auletta et al. '06] Agent can only report better speed.

Combinatorial auctions [Krysta, Ventre '10],
[Fotakis, Krysta, Ventre '13] Agent can understate his profit, but not overstate it.

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Local characterization of Truthfulness [M. Saks, L. Yu '05], [A. Archer, R. Kleinberg '08]

Definition

A social choice function $f : D \to O$ satisfies *local weak monotonicity (local WMON)* if for every $x \in D$ and every line L through x, there exists an open neighborhood U about x such that

$$(x - y) \cdot (f(x) - f(y) \geq 0$$

for all $y \in L \cap U$

Local characterization of Truthfulness [M. Saks, L. Yu '05], [A. Archer, R. Kleinberg '08]

Finite outcome space O

Theorem [M. Saks, L. Yu '05]

If $|O|$ is finite, D is convex, and f satisfies local WMON, then f is truthfully implementable with money.

Local characterization of Truthfulness [M. Saks, L. Yu '05], [A. Archer, R. Kleinberg '08]

Infinite outcome space O

Theorem [A. Archer, R. Kleinberg '08]

Let D be a convex space, and f be a locally path-integrable then f satisfies local WMON if and only if f is truthfully implementable with money.

M^ϵ Verification

Definition

If D is a convex domain then we define the misreport correspondence M^ϵ as follows :

$$\forall x \in D \quad M^\epsilon(x) = \{y \mid y \in D \ \wedge \ ||x - y|| \leq \epsilon \ \}$$

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Using the results of M.Saks, L. Yu and A. Archer, R. Kleinberg we can prove the following.

Theorem [Theorem 3.1 Caragiannis et. al]

For any $\epsilon > 0$ a social choice function f on a convex domain D is truthfully $M^\epsilon-$impementable if and only if it is truthfully implementable.

Contents

Gibbard-Satterthwaite

Theorem

In the case where $D = L(O)$ if f is a incetive compatible voting rule onto O where $|O| \geq 3$, then f is a *dictatorship*.

M^{swap} Verification

In the case of strategic voting we define the following partial verification.

Definition

If $D = L(O), |O| = m$ then we define the misreport correspondence M^{swap} as follows :

$$\forall R \in D \quad M^{swap}(R) = \{R(a_{j-1} \leftrightarrow a_j) : j = 2, 3, ..., m\}$$

M^{swap} Verification

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Theorem [Theorem 3.3 Caragiannis et. al]

If f is a $M^{swap}-$truthful voting rule onto O where $|O| \geq 3$, then f is a dictatorship.

Contents

Local to Global without money

Definition

A path P in G_M between to types x, y is called *order preserving* path if for every other type w in P

$$\forall a, b \in O \quad (x(a) > x(b) \wedge y(a) \geq y(b) \Rightarrow w(a) > w(b))$$

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Proposition

Let D be finite and let M be the correspondence of a *symmetric-order preserving* verification and f be a social choice function then f is $M-$truthfully implementable if and only if f is truthfully implementable.

Local to Global with money

Definition

A path P in G_M between to types x, y is called *difference preserving* path if for every other type w in P

$$\forall a, b \in O \;\; (x(a) - x(b) \geq w(a) - w(b) \geq y(a) - y(b) \vee$$

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Local to Global with money

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$$\vee \ y(a) - y(b) \geq w(a) - w(b) \geq x(a) - x(b))$$

Proposition

Let M be the correspondence of a symmetric-difference preserving verification and f be a social choice function then f is M-truthfully implementable if and only if f is truthfully implementable.